Methods of Applied Mathematics

Sheet 2. Scaling

1. In Example 1.15 of lectures three different ways of obtaining a dimensionless version of the projectile problem were given (Versions A, B and C). Verify that in each case the characteristic values used for the variables involved lead to the versions given.

2. If we assume that air-resistance gives a force proportional to speed and that the gravitational force does not vary with height, then (while travelling upwards) the projectile problem has the form:

$$h''(t) + kh'(t) + g = 0, \quad h(0) = 0, \; h'(0) = V,$$

where k is a positive constant and V is the initial upwards speed.
In the case where V is small, justify the use of V^2/g and V/g as characteristic values of length and time to be used for scaling h and t. With respect to what is V small?
Find appropriate characteristic values to deal with the early stages of the motion in the case where V is large and give their physical meanings.

3. Suppose that $[u] = [f(t)]$. In each of the following cases, use $f(t)$ to find an appropriate characteristic value u_c for u and a corresponding characteristic value t_c for t.

   (a) $f(t) = A\cos\lambda t \qquad t \in [0, \infty), \quad A, \lambda > 0;$

   (b) $f(t) = \exp(-at), \qquad t \in [0, \infty), \quad a > 0;$

   (c) $f(t) = 100\exp\{(1-t)/1000)\}, \qquad t \in [0, 1].$