Methods of Applied Mathematics

Sheet 1. The Pi Theorem

1. A ball of radius r and density ρ falls at constant speed v under the influence of gravity g (acceleration due to gravity) in a liquid of density σ and viscosity μ (mass per length per unit time). It is observed experimentally that
$$v = \frac{2r^2\rho g}{9\mu}\left(1 - \frac{\sigma}{\rho}\right).$$
Is this law unit-free?

2. Suppose that some physical phenomenon is described by a unit-free law of the form
$$f(P, l, m, t, \rho) = 0,$$
where P, l, m, t and ρ denote pressure, length, mass, time and density, respectively. Show that there is an equivalent physical law of the form
$$G(l^3\rho/m, t^6P^3/(m^2\rho)) = 0.$$
[*Hint*:- It is enough to show that the solution space of $AX = 0$ is 2-dimensional and that the vecors X_1 and X_2 corresponding to $\pi_1 = l^3\rho/m$ and $\pi_2 = t^6P^3/m^2\rho$ are basis vectors. Why is this enough?]

3. Use dimensional analysis to prove Pythagoras's theorem.
You may assume that
   - in a right-angled triangle, the area A is determined by the length c of the hypotenuse and the magnitude ϕ of the smallest internal angle,

   and
   - the perpendicular dropped from the vertex opposite the hypotenuse to the hypotenuse divides the triangle into two right-angled triangles, each of which is similar to the given triangle.