G1101 Random Processes 2008–09: Exercises 5

Epidemics

5.1 In a household of size 6, four have an infectious disease. The disease spreads through the household as a simple epidemic with rate $\beta = 1{\cdot}2$.

   (a) Find the probability distribution of the total number of infected people at time t.

   (b) Find the density of W, the duration of the epidemic.

   (c) Find the mean of W.

5.2 Consider a simple epidemic when the population size $n + 1$ is an odd integer, i.e. n is even.

   (a) Show that the expected duration of the epidemic if $y_0 = 1$ is exactly twice that if $y_0 = \frac{1}{2}n + 1$.

   (b) When $y_0 = 1$, is the variance of the epidemic length greater than, equal to, or less than twice that when $y_0 = \frac{1}{2}n + 1$?

5.3 Consider the general epidemic when $x_0 = 2$ infectives, $y_0 = 2$ susceptibles and $\rho = 1{\cdot}5$.

   (a) Show on a diagram the possible transitions and their associated probabilities.

   (b) Find the survivor probability distribution and hence the expected value of $X(\infty)$.

5.4 For the general epidemic when $x_0 = 2$, $y_0 = n-1$, find the survivor probability distribution for $\rho = 1$.

5.5 A town has a population of $10\,000$ people. Two people move to the town with a disease that the whole population is susceptible to. Suppose that on average everybody meets 12 other people per day and that the average duration of the disease is just four hours.

   (a) What is the approximate probability that there is a major outbreak of the disease?

   (b) What would this probability be if on average everybody met only 5 other people per day?