Queues

3.1 Specify the queues, in the notation A/B/k, that correspond to the following scenarios. In each case, use the general criterion for stability $\lambda < n\mu$ to determine the stability of the queue.

(a) A medical practice with four doctors, who have appointments under which patients arrive exactly 3 minutes apart, the patient at the head of the queue seeing the first available doctor. Consultations are such that however long a consultation has been in process, the remaining consultation time has the same distribution with mean 10 minutes.

(b) A barber's shop, with two assistants; customers arrive at random at rate one every 12 minutes, haircuts take between 15 and 25 minutes uniformly.

(c) A post office with one server. Customers arrive at random at rate 18 per hour; 20% buy stamps (taking uniformly 30 to 45 seconds); 30% post a parcel (taking uniformly 2 to 3 minutes); the rest have more complicated transactions taking uniformly between 3 and 6 minutes.

3.2 For a simple queue with arrival rate 5 per hour and average service time ten minutes, after a long period of time

(a) What is the expected length of the queue?

(b) What is the probability that the queue is empty?

(c) What is the probability that there are more than 10 people in the queue?

(d) How long is the average busy period?

(e) What is the expected number of customers served during a busy period?

3.3 There are two banks on the High Street. One has one teller on duty; the other has two. Everybody knows this, and the arrival rate of customers at the second bank is twice that of the first. Which bank should you visit in order to maximise the probability of being served immediately? (Assume for simplicity that service times are exponentially distributed.)

3.4 Bank customers arrive at an ATM (Automatic Teller Machine) at rate 1 every 2 minutes. It is assumed that it takes almost exactly 1 minute 20 seconds for each customer to be served.

(a) What is the probability that the machine is free when a customer arrives?

(b) What is the expected number of people in the queue (including anyone using the ATM) when a customer arrives?

(c) How long, on average, would a customer expect to wait before beginning to use the ATM if there were six people ahead in the queue (including the one using the machine)?

(d) How long, on average, would a customer expect to wait before being able to begin using the ATM?

3.5 For a queue where customers arrive as a Poisson process, at rate 1/3, establish whether the queue is stable and, if so, calculate the equilibrium expected queue length, when the service time is

(a) Exponential, parameter ½;

continued ...

(b) Uniform on the interval (1,4);
(c) distributed as $4Z^2$, where Z is standard Normal;
(d) Pareto, with density $f(x) = 2x^{-3}$ for $x > 1$.

3.6 Suppose that for the simple queue, arrivals occur at rate λ, but the probability that a new arrival stays if there are x people already in the queue (including being served) is α^x, where $0 < \alpha < 1$. Show that the equilibrium distribution has probability function p_x which is proportional to

$$\rho^x \alpha^{x(x-1)/2} \qquad (x = 0, 1, 2, \ldots),$$

where $\rho = \lambda/\mu$.

For what values of ρ does this distribution exist?

3.7 For the simple queue with finite waiting room of capacity $c+1$ persons (§4.9 of the lecture notes), what is the equilibrium distribution for the case when $\rho = 1$, i.e. $\lambda = \mu$?